Mathematical Modeling and Simulation on Inter-Orbit-Links in Treble-Layer Satellite Network

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***Abstract*— The Intra-Orbit-Links(IOL), which play the key role during the satellite communication, is introduced. The geometrical model of IOL in the Treble-Layer Satellite Network(TLSN) is constructed. The IOL inside different kinds of TLSN is simulated, that not only the advice of research on More-Layer Satellite Network is given, but the conclusion that a MEO-GEO-IGSO TLSN is batter than kinds of constellation construction is also proved.**

I. Introduction

As the discussion of IOL be in progress during the current period, the structure of Inter-Orbit-Links (IOL) is more complicated[1], so it is necessary to analyze the IOL between a treble-layer satellite network.

Wang Zhen-yong and Wang Ke-feng discussed the IOL between MEO layer and LEO layer in paper [2] and [3] in 2006, respectively. Till know, there is no papers discussed the Treble-Layer IOL. During this paper, we will discuss four different kinds of TLSNs. Based on the common satellite types, we choose LEO satellite, MEO satellite, GEO satellite and IGSO satellite.

II. Geometrical Analysis

A. IOL Between LEO/MEO Layer and GEO Layer

In *Figure.1*, O is the center of the earth, S_1 and S_2 denote the positions of a LEO/MEO satellite and a GEO satellite respectively. The IOL connecting LEO/MEO satellite S_1 and GEO satellite S_2 is represented by IOL(S_1_S_2). S_1' and S_2' are the projection points of S_1 and S_2 on the earth surface, so S_1' and S_2' have the same longitudes and latitudes as S_1 and S_2. ϕ is the angle between beeline OS_1 and beeline OS_2.

Based on motion equation of circular orbit satellites in the earth's center inertial frame, longitude λ_1 and latitude ϕ_1 of satellite points of LEO/MEO satellite S_1 is achieved below by geometric analysis:

$$\begin{cases} \lambda_1 = \dot{\lambda}_1 + \arctan\big(\cos i_1 \tan(\omega_1 t + \gamma_1)\big) - \omega_e t, \\ \phi_1 = \arcsin\big(\sin i_1 \sin(\omega_1 t + \gamma_1)\big), \end{cases} \tag{1}$$

where t is the time that satellites pass ascending node, i_1 is the inclination of the orbit of S_1, $\dot{\lambda}_1$ is the longitude of ascending node of S_1, ω_1 is the angular velocities of S_1, ω_e is the rotation angular velocities of earth, γ_1 is the original phase in the orbit of S_1, R_e is the radius of the earth, R_1 and R_2(constant) are the altitudes of LEO/MEO layer and GEO layer. For the longitude λ_2 and latitude ϕ_2 of GEO satellite S_2, we have $\phi_2 = 0$ and λ_2(constant).

With geometric analysis, the distance $r_{1,2}$ between S_1 and S_2 can be achieved from the equation below:

$$\begin{aligned} r_{1,2}^2 &= (R_1 + R_e)^2 + (R_2 + R_e)^2 \\ &\quad -2\cos\phi(R_1 + R_e)(R_2 + R_e), \end{aligned} \tag{2}$$

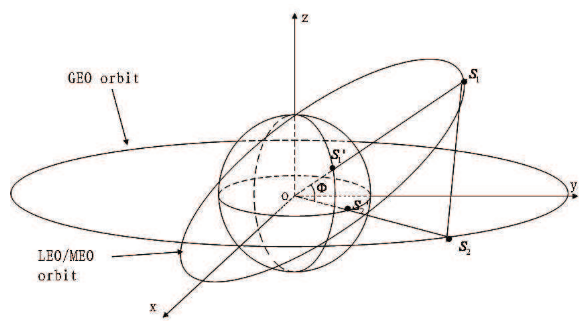

Figure.1: IOL between LEO/MEO and GEO

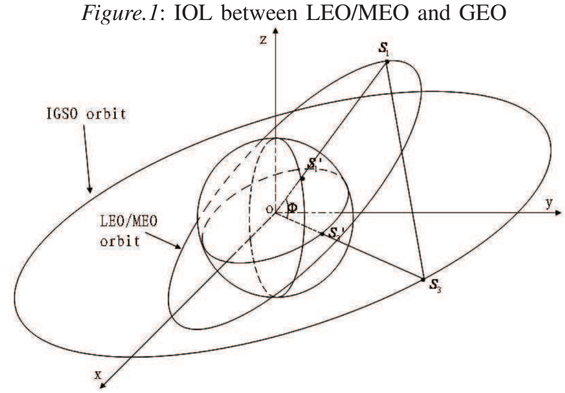

Figure.2: IOL between LEO/MEO and IGSO

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B. IOL Between LEO/MEO Layer and IGSO Layer

In *Figure.2*, O is the center of the earth, S_1 and S_3 denote the positions of a LEO/MEO satellite and a IGSO satellite respectively. The IOL connecting LEO/MEO satellite S_1 and IGSO satellite S_3 is represented by IOL($S_1\_S_3$). S_1' and S_3' are the projection points of S_1 and S_3 on the earth surface, so S_1' and S_3' have the same longitudes and latitudes as S_1 and S_3. ϕ is the angle between beeline OS_1 and beeline OS_3.

Based on motion equation of circular orbit satellites in the earth inertial frame, longitude λ_1 and latitude ϕ_1 of satellite points of LEO/MEO satellite S_1 is given by equation (1). Longitude λ_3 and latitude ϕ_3 of satellite points of IGSO satellite S_3 is achieved below by geometric analysis:

$$\begin{cases} \lambda_3 = \dot{\lambda}_3 + \arctan\left(\cos i_3 \tan(\omega_3 t + \gamma_3)\right) - \omega_e t, \\ \phi_3 = \arcsin\left(\sin i_3 \sin(\omega_3 t + \gamma_3)\right), \end{cases} \tag{3}$$

where t is the time that satellites pass ascending node, i_3 is the inclination of the orbit of S_3, $\dot{\lambda}_3$ is the longitude of ascending node of S_3, ω_3 is the angular velocities of S_3, ω_e is the rotation angular velocities of earth, γ_3 is the original phase in the orbit of S_3, R_e is the radius of the earth, R_3(constant) is the altitudes of IGSO layer.

With geometric analysis, the distance $r_{1,3}$ between S_1 and S_3 can be achieved from the equation below:

$$\begin{aligned} r_{1,3}^2 &= (R_1 + R_e)^2 + (R_3 + R_e)^2 \\ &\quad -2\cos\phi(R_1 + R_e)(R_3 + R_e). \end{aligned} \tag{4}$$

C. IOL Between GEO and IGSO

In this case, the position of satellites S_2 and S_3 is similar to the MEO and GEO case. In *Figure.3*, O is the center of the earth, S_2 and S_3 denote the positions of a GEO satellite and a IGSO satellite respectively. The IOL connecting GEO satellite S_2 and IGSO satellite S_3 is represented by IOL($S_2\_S_3$). S_2' and S_3' are the projection points of S_2 and S_3 on the earth surface, so S_2' and S_3' have the same longitudes and latitudes as S_2 and S_3. ϕ is the angle between beeline OS_2 and beeline OS_3.

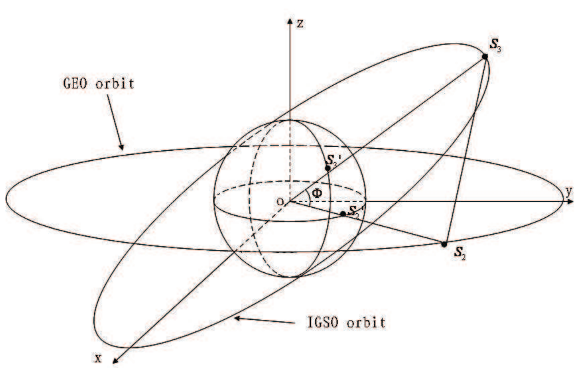

Figure.3: IOL between GEO and IGSO

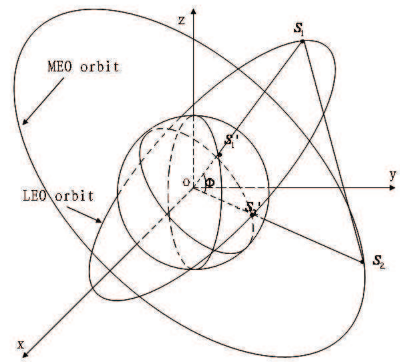

Figure.4: IOL between MEO and LEO

Based on motion equation of circular orbit satellites in the earth inertial frame, longitude λ_2 and latitude ϕ_2 of satellite points of GEO satellite S_2 is given by equation (1); longitude λ_3 and latitude ϕ_3 of satellite points of IGSO satellite S_3 is given by equation (3). So, the distance $r_{2,3}$ between S_2 and S_3 can be achieved from the equation below:

$$\begin{aligned} r_{2,3}^2 &= (R_2 + R_e)^2 + (R_3 + R_e)^2 \\ &\quad -2\cos\phi(R_2 + R_e)(R_3 + R_e). \end{aligned} \tag{5}$$

D. IOL Between MEO Layer and LEO Layer

In *Figure.4*, O is the center of the earth, S_1 and S_2 denote the positions of a MEO satellite and a LEO satellite respectively. The IOL connecting MEO satellite S_1 and LEO satellite S_2 is represented by IOL($S_1\_S_2$). S_1' and S_2' are the projection points of S_1 and S_2 on the earth surface, so S_1' and S_2' have the same longitudes and latitudes as S_1 and S_2. ϕ is the angle between beeline OS_1 and beeline OS_2.

Based on motion equation of circular orbit satellites in the earth's center inertial frame, longitude λ_1 and latitude ϕ_1 of satellite points of LEO/MEO satellite $S_j (j = 1, 2)$ is achieved below by geometric analysis:

$$\begin{cases} \lambda_j = \dot{\lambda}_j + \arctan\left(\cos i_j \tan(\omega_j t + \gamma_j)\right) - \omega_e t, \\ \phi_j = \arcsin\left(\sin i_j \sin(\omega_j t + \gamma_j)\right), \end{cases} \tag{6}$$

where t is the time that satellites pass ascending node, i_j is the inclination of the orbit of S_j, $\dot{\lambda}_j$ is the longitude of ascending node of S_j, ω_j is the angular velocities of S_j, ω_e is the rotation angular velocities of earth, γ_j is the original phase in the orbit of S_j, R_e is the radius of the earth, R_j is the altitudes of LEO/MEO layer.

With geometric analysis, the distance $r_{1,2}$ between S_1 and S_2 can be achieved from the equation below:

$$\begin{aligned} r_{1,2}^2 &= (R_1 + R_e)^2 + (R_2 + R_e)^2 \\ &\quad -2\cos\phi(R_1 + R_e)(R_2 + R_e). \end{aligned} \tag{7}$$

E. The Azimuth and Elevation of IOL

For the convenience while calculating, we will use the earth's center inertial frame. The transform from earth inertial frame (ϕ, λ, h) to earth's center inertial frame (X, Y, Z) is:

$$\begin{cases} X = (N+h)\cos\phi\cos\lambda, \\ Y = (N+h)\cos\phi\sin\lambda, \\ Z = \big(N(1-e^2)+h\big)\sin\phi, \end{cases} \tag{8}$$

where N denotes the radius of curvature in prime vertical of earth, e denotes the first eccentricity ratio of the earth.

To characterization the geometrical position, we need the azimuth and elevation of IOL, which shown in *Figure.5*. Let S_i and S_j be the positions of two different kinds of satellites, S_i^0 and S_j^0 are the projections of S_i and S_j on plane OXY, S_{ij} be the node of beeline S_iS_j and plane OXY, θ_a, θ_e is the azimuth and elevation of S_iS_j, respectively. Thus,

$$\begin{cases} \theta_a = \arctan\frac{y_i-y_j}{x_i-x_j} \\ \theta_e = \arctan\frac{z_i-z_j}{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}. \end{cases} \tag{9}$$

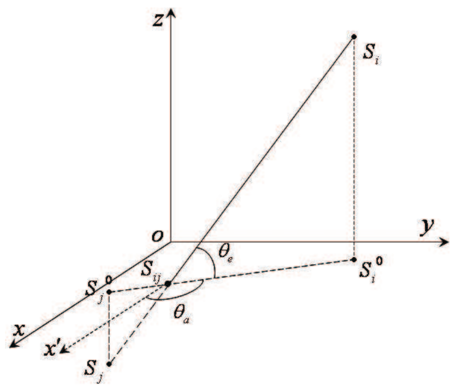

Figure.5: The azimuth and elevation of IOL

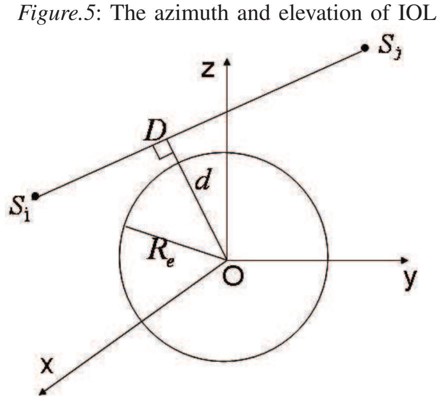

Figure.6: Connective IOL when $d \geq R_e$

III. Connectivity of IOL

Treble-Layer IOL is not always connective because the earth and its aerosphere may shut it off. Consider the obstruction to the IOL of the earth and its aerosphere, the distance of IOL have a maximal distance.

The geometrical analysis is shown as *Figure.6*. Where d denote the distance between the earth's core O and the IOL S_iS_j, where $(i, j = 1, 2, 3$ and $i \neq j)$. When $d \geq R_e$, the IOL is connective.

$$\begin{aligned} d^2 &= x_i^2 + y_i^2 + z_i^2 \\ &\quad - \frac{\big(x_i(x_j-x_i) + y_i(y_j-y_i) + z_i(z_j-z_i)\big)^2}{(x_j-x_i)^2+(y_j-y_i)^2+(z_j-z_i)^2} \\ &\geq R_e^2. \end{aligned} \tag{10}$$

While $d < R_e$, there is only one case that the IOL is still connective which is shown as *Figure.7*. In this case, $\angle S_jS_iO$ is an obtuse angle, and $\cos\angle S_jS_iO < 0$, meanwhile. That is to say:

$$\begin{aligned} & \cos\angle S_2S_1O = \tfrac{|OS_i|^2+|S_iS_j|^2-|OS_j|^2}{2(|OS_i|+|S_iS_j|)} < 0 \\ \Rightarrow \quad & |OS_i|^2 + |S_iS_j|^2 < |OS_j|^2, \end{aligned}$$

namely:

$$\begin{aligned} & x_i^2 + y_i^2 + z_i^2 + (x_j-x_i)^2 + (y_j-y_i)^2 + (z_j-z_i)^2 \\ < \quad & x_j^2 + y_j^2 + z_j^2. \end{aligned} \tag{11}$$

In the cases out of the two figures above, the IOL is surely disconnected. By formula (10) and (11), the connectivity of IOL can easily be ensure by computer. In the next section, we will choose different kinds of satellites and simulate the IOL between them in order to give an intuitionistic impressions.

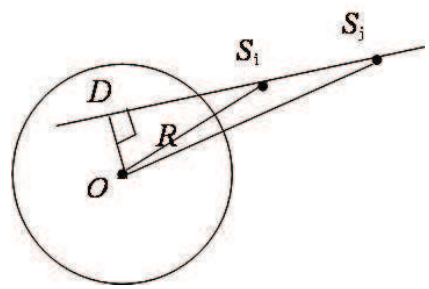

Figure.7: Connective IOL when $d < R_e$

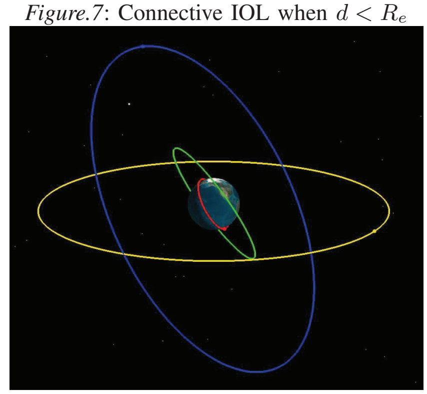

Figure.8: Construction of Satellites

IV. Simulation and Analysis

A. Simulation of IOL

While simulating, the satellites chosen is shown in *Figure.8*. The parameters and geometrical positions of the satellites we choose are given as the two tables below:

Table 1. Parameters of Satellites				
Satellite	LEO	MEO	GEO	IGSO
$h(km)$	276.875	10000.141	35785.726	35788.231
$i(^\circ)$	60	60	0	50
$\dot{\lambda}(^\circ)$	110(W)	150(E)	-	100(E)
T(s)	5400	20859	86164	86164

Table 2. Original Positions of Satellites

Satellite	$\gamma(\phi(^\circ), \lambda(^\circ), h(km))$
LEO	(0.023,-70.251,274.419)
MEO	(-0.019,150,10000)
GEO	(0.023,-80,35786.032)
IGSO	(-0.023,100,35786.032)

Where h denotes the orbit altitude of satellite, i denotes the orbit inclination angle, $\dot{\lambda}$ denotes the longitude of ascending node, T denotes the revolution periods, γ denotes the original position.

While simulating, the parameters of the given LEO, MEO, GEO and IGSO are shown in *Table 1*, the whole simulation period is 2 days. The simulation results are exhibited by the figures which shown from *Figure.9* to *Figure.14*. From these pictures we can see clearly the variational rules of the distance, azimuth and elevation of IOL, also can find the time when the IOLs are disconnected. Thus, the goal of doing this simulation is not only give a depiction to the TLSN, but also ascertain one better construction from the fore given constructions.

The following part is going to simulate just two satellites from the above table.

1) Simulation of IOL between LEO and MEO:

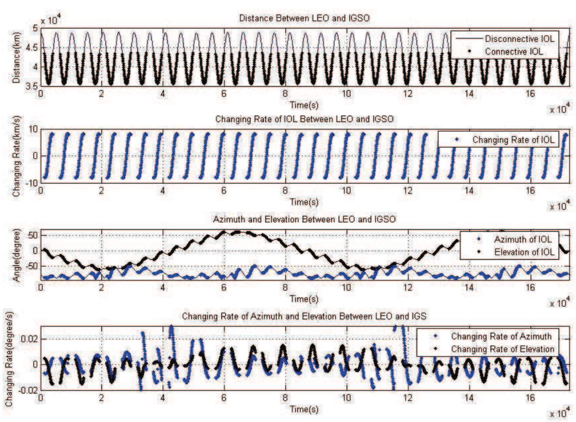

Figure.9: Simulation of IOL between LEO and MEO

2) Simulation of IOL between LEO and GEO:

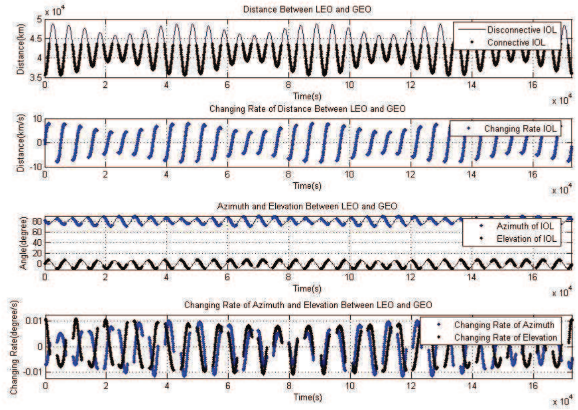

Figure.10: Simulation of IOL between LEO and GEO

3) Simulation of IOL between LEO and IGSO:

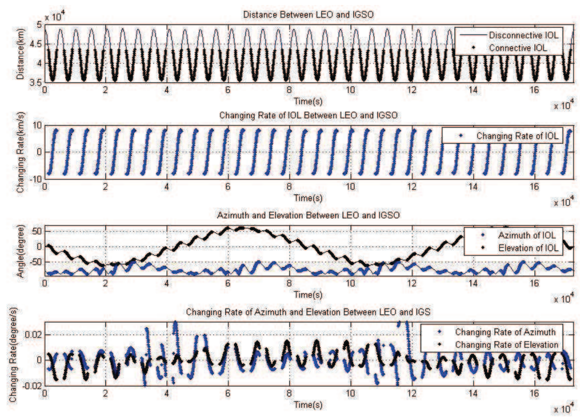

Figure.11: Simulation of IOL between LEO and IGSO

4) Simulation of IOL between MEO and GEO:

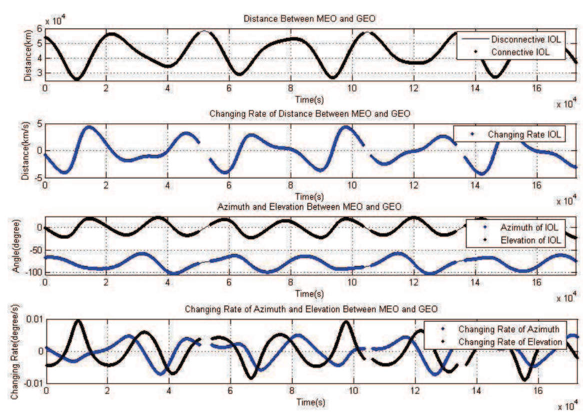

Figure.12: Simulation of IOL between MEO and GEO

5) Simulation of IOL between MEO and IGSO:

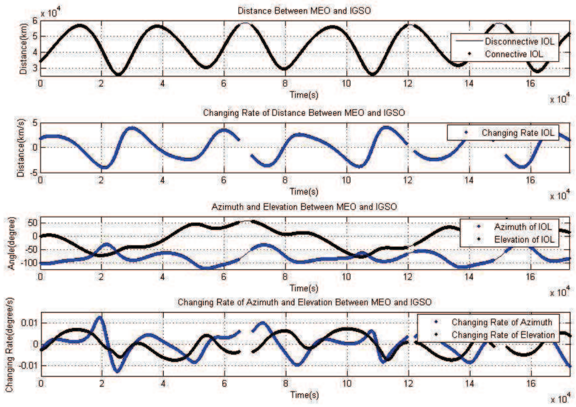

Figure.13: Simulation of IOL between MEO and IGSO

6) *Simulation of IOL between GEO and IGSO:*

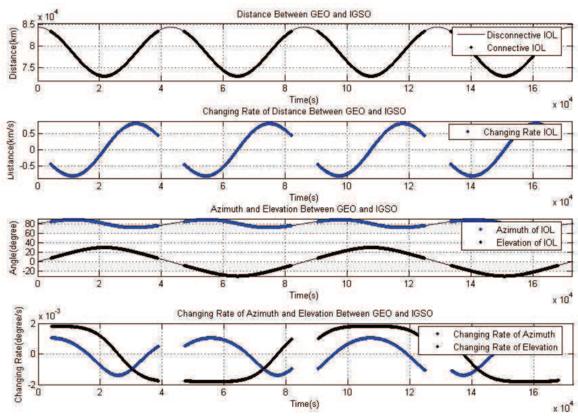

Figure.14: Simulation of IOL between GEO and IGSO

B. Connective Times of TLSN

We choose three satellites from the former discussion to build a TLSN and analysis the qualities. The distance, azimuth and elevation about each different TLSN will be ascertain by section IV-A. Thus, to discuss any kind of TLSN, we only need to choose the corresponding simulation results. Here, without the background of any realistic constellation, which we care about is the time when this TLSN is able to use. That is to say, we concern about two questions:

1). When any IOL of this TLSN is able to use?

2). When at least two of the IOLs inside this TLSN is able to use?

To attain these goals, it is need to clarify when the IOL between two satellites is connective.

1) *LEO-MEO-GEO TLSN:* Consider the TLSN make up with LEO, MEO, GEO satellites, the time that whether each IOL is connective is shown in *Figure.15*.

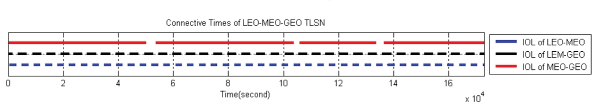

Figure.15: Connective Times of LEO-MEO-GEO TLSN

2) *LEO-MEO-IGSO TLSN:* Consider the TLSN make up with LEO, MEO, IGSO satellites, the time that whether each IOL is connective is shown in *Figure.16*.

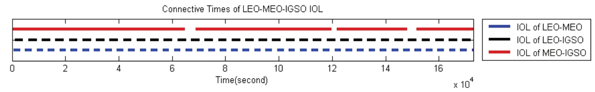

Figure.16: Connective Times of LEO-MEO-IGSO TLSN

3) *LEO-GEO-IGSO TLSN:* Consider the TLSN make up with LEO, GEO, IGSO satellites, the time that whether each IOL is connective is shown in *Figure.17*.

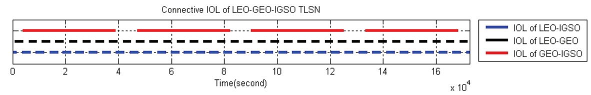

Figure.17: Connective Times of LEO-GEO-IGSO TLSN

4) *MEO-GEO-IGSO TLSN:* Consider the TLSN make up with MEO, GEO, IGSO satellites, the time that whether each IOL is connective is shown in *Figure.18*.

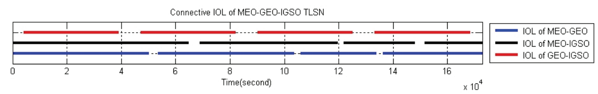

Figure.18: Connective Times of MEO-GEO-IGSO TLSN

C. Compare and Analysis

Now we will answer the two questions we concern about which given in section IV-B.

To answer these questions, some statistical data is necessarily:

1). The total time periods when each IOL in the given TLSN is connective: T_3;

2). The maximal(or minimal) continual time period when each IOL in the given TLSN is connective: $Max(T_3)$(or $Min(T_3)$);

3). The average time period when each IOL is connective: $\bar{T}_3$;

4). The total time periods when two IOLs in the given TLSN are connective: T_2;

5). The maximal(or minimal) continual time period that two IOLs in the given TLSN is connective: $Max(T_2)$(or $Min(T_2)$);

6). The average time period when two IOLs are connective: $\bar{T}_2$;

7). The total time periods when the given TLSN is disconnective: T_{dis}.

The results are shown in *Table 3*(the time unit is minute).

Table 3. Parameters of Satellites

TLSN	LMG-T	LMI-T	LGI-T	MGI-T
T_3	732	725	265	1965
$Max(T_3)$	33	33	10	430
$Min(T_3)$	3	3	1	1
$\bar{T}_3$	20.333	21.324	6.795	178.636
T_2	2135	2149	2615	915
$Max(T_2)$	110	180	212	140
$Min(T_2)$	31	21	32	3
$\bar{T}_2$	53.375	58.081	62.262	83.182
T_{dis}	13	6	0	0

Where "LMG-T", "LMI-T", "LGI-T", and "MGI-T" are denote the "LEO-MEO-GEO TLSN", "LEO-MEO-IGSO TLSN", "LEO-GEO-IGSO TLSN" and "MEO-GEO-IGSO TLSN" respectively.

From *Table 3*, it is clear that the MEO-GEO-IGSO TLSN is a batter choice based on the satellites given in *Table 1* and *Table 2*. Because $\bar{T}_3$ and $\bar{T}_2$ shows the stability of one TLSN, which the value is more bigger the TLSN is more stable.

V. CONCLUSION

A TLSN is proposed to support broadband multimedia service for future global satellite communication systems. The network topology, functions of each layer and constellations are considered. Geometric characteristics and the connectivity of IOLs are analyzed. By simulations, geometric characteristics of IOL and results of the connectivity are given about, which shows that the IOL should be maintained according to the constraints of pointing elevation and length instead of the in sight principle. The simulation results also show that the MEO-GEO-IGSO TLSN is more stable the other kinds of TLSNs, thus, in order to use the TLSN in the civil fields, military fields or other places, a batter choice is to built the MEO-GEO-IGSO TLSN in the future constellation.

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